

# What do WMAP and SDSS really tell about inflation?

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We derive new constraints on the Hubble function  $H(\phi)$  and subsequently on the inflationary potential  $V(\phi)$  from WMAP 3-year data combined with the Sloan Luminous Red Galaxy survey (SDSS-LRG), using a new methodology which appears to be more generic, conservative and model-independent than in most of the recent literature, since it depends neither on the slow-roll approximation for computing the primordial spectra, nor on any extrapolation scheme for the potential beyond the observable e-fold range, nor on additional assumptions about initial conditions for the inflaton velocity. This last feature represents the main improvement of this work, and is made possible by the reconstruction of  $H(\phi)$  prior to  $V(\phi)$ . Our results only rely on the assumption that within the observable range, corresponding to  $\sim 10$  e-folds, inflation is not interrupted and the function  $H(\phi)$  is smooth enough for being Taylor-expanded at order one, two or three. We conclude that the variety of potentials allowed by the data is still large. However, it is clear that the first two slow-roll parameters are really small while the validity of the slow-roll expansion beyond them is not established.

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Cosmic inflation was introduced as a simple and aesthetically elegant scenario of the early Universe evolution which is capable of explaining its main properties observed at the present time [1, 2, 3, 4, 5, 6]. As a very important byproduct it provides a successful mechanism for the quantum-gravitational generation of primordial scalar (density) perturbations and gravitational waves [7, 8, 9, 10, 11, 12, 13]. The Fourier power spectrum  $\mathcal{P}_{\mathcal{R}}(k)$  of the former ones is observed today in the cosmic microwave background (CMB) and the large scale structure (LSS). Vice versa, at present the CMB and the LSS provide the only quantifiable observables which can confirm or falsify inflationary predictions. That is why matching concrete inflationary models to observations has become one of the leading quests in cosmology.

In the simplest class of inflationary models, inflation is driven by a single scalar field  $\phi$  (an inflaton) with some potential  $V(\phi)$  which is minimally coupled to the Einstein gravity. For these models, some new conservative bounds on  $V(\phi)$  were presented recently in [14]. Until then, most post-WMAP3 studies concerning  $V(\phi)$  relied on the slow-roll approximation in the calculation of perturbation power spectra and their relation to values of  $\phi$  during inflation [15, 16, 17, 18, 19, 20, 21, 22, 23], or made an extrapolation of  $V(\phi)$  from the observable window till the end of inflation [24, 25, 26] (a numerical integration of exact wave equations for perturbations to obtain primordial power spectra was also performed in

Refs. [27, 28, 29] for specific inflationary models). The extrapolation over the full duration of inflation is more constraining than the data alone. Instead, Ref. [14] focused only on the observable part of the potential to see up to what extent current data really constrains inflation.

For this class of models, the evolution of a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe can be described by [30, 31]

$$\dot{\phi} = -\frac{m_P^2}{4\pi} H'(\phi) \quad (1)$$

$$-\frac{32\pi^2}{m_P^4} V(\phi) = [H'(\phi)]^2 - \frac{12\pi}{m_P^2} H^2(\phi). \quad (2)$$

whenever  $\dot{\phi} \neq 0$  and not specifically during inflation (so  $H'(\phi) \neq 0$ , too). Here  $H(\phi(t)) \equiv \dot{a}/a$ ,  $a(t)$  is the FLRW scale factor, a dot denotes the derivative with respect to the cosmic time  $t$ , a prime with respect to an argument, and we have set  $Gm_P^2 = \hbar = c = 1$ . If  $V(\phi)$  is considered as the defining quantity, the initial conditions for generating the observable window are determined by the set  $\{\phi_{\text{ini}}, V(\phi)\}$ . In Ref. [14], the inflaton potential was parametrized as a Taylor expansion up to some order, to see up to what extent the potential can be constrained by pure observations. However, in order to reduce the number of free parameters,  $\dot{\phi}_{\text{ini}}$  was fixed for each model by demanding that the inflaton follows its attractor solution just when the observable modes exit the horizon. In practice this means that the results of Ref. [14] assumed that inflation started *at least* a few e-folds before the observable modes left the horizon. These precurring e-folds led to a slightly stronger bound on the potentials than the data itself could actually give, although this extra constraining power stands in no proportion to an extrapolation over the full duration of inflation.

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Eqs. (1, 2) however show that when one considers  $H(\phi)$  as the defining quantity, all initial conditions are already uniquely set by  $H(\phi)$ . Moreover, the slow-roll conditions which require, in particular, that the first term in the rhs of Eq. (2) is much less than the last one need not be imposed *ab initio*. In this Letter we derive the bounds on  $H(\phi)$  during observable inflation using its Taylor expansion at various orders. We infer for this some constraints on  $V(\phi)$  under an even more conservative approach than in Ref. [14], since the present method requires absolutely no extrapolation outside of the observable region (either forward or backward in time). Our only restriction is to assume that observable cosmological perturbations originate from the quantum fluctuations of a single inflaton field, which dynamics during observable inflation is compatible with a smooth, featureless  $H(\phi)$ .

*Method.* We used the publicly available code COSMOMC [32] to do a Monte Carlo Markov Chain (MCMC) simulation. We added a new module (released at <http://wwwlapp.in2p3.fr/~valkenbu/inflationH/>) which computes numerically the primordial spectrum of scalar and tensor perturbations for each given function  $H(\phi - \phi_*)$ , where  $\phi_*$  is an arbitrary pivot scale in field space. This module is simpler than the one in Ref. [14], since the code never needs to find an attractor solution of the form  $\dot{\phi}(\phi)$ . The comoving pivot wavenumber is fixed once and for all to be  $k_* = 0.01 \text{ Mpc}^{-1}$ , roughly in the middle of the observable range. Primordial power spectra are computed in the range  $[k_{\min}, k_{\max}] = [5 \times 10^{-6}, 5] \text{ Mpc}^{-1}$  needed by CAMB, imposing that  $k_*$  leaves the Hubble radius when  $\phi = \phi_*$ . In practice, this just means that for each model the code normalizes the scale factor to the value  $a_* = k_*/H_*$  when  $\phi = \phi_*$ . Note that by mapping a window of inflation to a window of observations today, our approach is independent of the mechanism of reheating. The evolution of each scalar/tensor mode is given by

$$\frac{d^2 \xi_{S,T}}{d\eta^2} + \left[ k^2 - \frac{1}{z_{S,T}} \frac{d^2 z_{S,T}}{d\eta^2} \right] \xi_{S,T} = 0 \quad (3)$$

with  $\eta = \int dt/a(t)$  and  $z_S = a\dot{\phi}/H$  for scalars,  $z_T = a$  for tensors. The code integrates this equation starting from the initial condition  $\xi_{S,T} = e^{-ik\eta}/\sqrt{2k}$  when  $k/aH = 50$ , and stops when the expression for the observed scalar/tensor power spectrum freezes out in the long-wavelength regime. More precisely, the spectra are given by

$$\frac{k^3}{2\pi^2} \frac{|\xi_S|^2}{z_S^2} \rightarrow \mathcal{P}_{\mathcal{R}}, \quad \frac{32k^3}{\pi m_P^2} \frac{|\xi_T|^2}{z_T^2} \rightarrow \mathcal{P}_h, \quad (4)$$

and integration stops when  $[d \ln \mathcal{P}_{\mathcal{R},h}/d \ln a] < 10^{-3}$ . If for a given function  $H(\phi - \phi_*)$  the product  $aH$  cannot grow enough for fulfilling the above conditions, the model is rejected. In addition, we impose that  $aH$  grows monotonically, which is equivalent to saying that inflation

Parameter	$n = 1$	$n = 2$	$n = 3$
$\Omega_b h^2$	$0.023 \pm 0.001$	$0.023 \pm 0.001$	$0.022 \pm 0.001$
$\Omega_{cdm} h^2$	$0.109 \pm 0.004$	$0.109 \pm 0.004$	$0.110 \pm 0.004$
$\theta$	$1.042 \pm 0.003$	$1.041 \pm 0.004$	$1.040 \pm 0.004$
$\tau$	$0.08 \pm 0.03$	$0.08 \pm 0.03$	$0.09 \pm 0.03$
$\ln \left[ \frac{4H_*^4}{H_*'^2 m_P^6} 10^{10} \right]$	$3.07 \pm 0.06$	$3.07 \pm 0.06$	$3.09 \pm 0.06$
$\left( \frac{H_*'}{H_*} \right)^2 m_P^2$	$0.079 \pm 0.031$	$0.072 \pm 0.056$	$0.081 \pm 0.067$
$\frac{H_*''}{H_*} m_P^2$	0	$-0.035 \pm 0.199$	$-0.079 \pm 0.247$
$\frac{H_*'''}{H_*} \frac{H_*'}{H_*} m_P^4$	0	0	$1.53 \pm 1.23$
$-\ln \mathcal{L}_{\max}$	1781.7	1781.4	1780.1

TABLE I: Bayesian 68% confidence limits for  $\Lambda$ CDM inflationary models with a Taylor expansion of  $H(\phi - \phi_*)$  at order  $n = 1, 2, 3$  (with the primordial spectra computed numerically). The last line shows the maximum likelihood. The first four parameters have standard definitions (see e.g. [14]).

is not interrupted during the observable range. If these conditions are satisfied, the power spectra are compared to observations.

We choose to parametrize  $H$  as a Taylor expansion with respect to  $\phi - \phi_*$  up to a given order  $n$  varying between one and three (this choice of background parametrization is equivalent to that in Ref. [18], as long as no extrapolation is made). Note that for  $n > 1$  such an assumption excludes  $\dot{\phi}$  and  $H'$  becoming zero at some value  $\phi = \phi_1$  in the range involved since then  $H(\phi)$  would acquire a non-analytic part beginning from the term  $\propto |\phi - \phi_1|^{3/2}$  (with  $V(\phi)$  being totally analytic at this point) [41]. As a cosmological background we used the standard  $\Lambda$ CDM-model with the free parameters shown in Table I.

*Results for  $H(\phi - \phi_*)$ .* In Fig. 1 we show the probability distribution of each parameter marginalized over the other parameters. The corresponding 68% confidence limits are displayed in Table I, as well as the minimum of the effective  $\chi^2$  for each model. This minimum does not decrease significantly when  $n$  increases, which reflects the fact that current data are compatible with the simplest spectra and potentials, but derivatives up to  $H'''$  can be constrained with good accuracy. Note that it would be very difficult to give bounds directly on the set  $\{H, H', H'', H''', \dots\}$ : indeed, these parameters are strongly correlated by the data, because physical effects in the power spectra depend on combinations of them. For example, at the pivot scale, the scalar amplitude is mainly determined by  $(H_*^2/H_*')^2$  and the tensor-to-scalar ratio  $r \equiv \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$  by  $(H_*'/H_*)^2$ . The scalar tilt  $n_S$  further depends on  $H_*''/H_*$ , and the scalar running on  $H_*'''H_*'/H_*^2$ . The Markov Chains can converge in a reasonable amount of time only if the basis of parameters (receiving flat priors) consists in functions of each of the above quantities, or linear combinations of them. However, we also show in the last plot of Fig. 1 the distri-

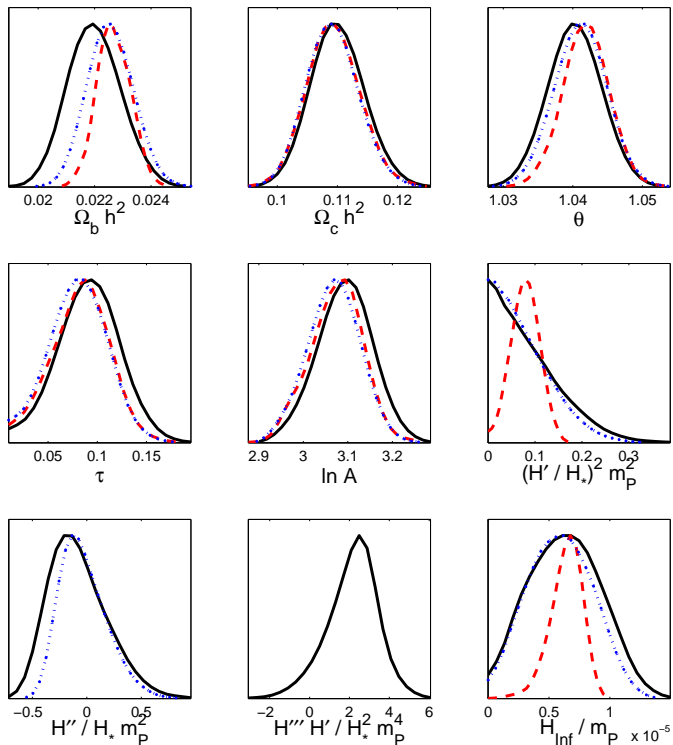


FIG. 1: Probability distribution for the eight independent parameters of the models considered here, normalized to a common arbitrary value of  $\mathcal{P}_{\max}$ . The ninth plot shows a related parameter (with non-flat prior): namely, the value of the expansion rate when the pivot scale leaves the horizon during inflation. Our three runs  $n = 1, 2, 3$  correspond respectively to the dashed red, dotted blue and solid black lines. The data consists of the WMAP 3-year results [15, 33, 34, 35] and the SDSS LRG spectrum [36]. The first four parameters have standard definitions (see e.g. [14]), and  $\ln A$  is a shortcut notation for the parameter defined in the fifth line of Table I.

bution of  $H_*$ : this information is useful since the energy scale of inflation is given by  $\lambda = (3H_*^2 m_P^2 / 8\pi)^{1/4}$ , but the displayed probability should be interpreted with care since this parameter has a non-flat prior.

The run  $n = 1$  is not very interesting. Indeed, imposing  $H''$  and higher derivatives to vanish leads to a one-to-one correspondence (at least in the slow-roll limit) between the amplitude and the tilt of the scalar spectrum. This feature is rather artificial and unmotivated. It explains anyway why the parameter  $H_*$  has exceptionally a lower bound in the  $n = 1$  case [42]. Much more interesting is the  $n = 2$  case for which the tensor ratio, scalar amplitude and scalar tilt are completely independent of each other, and the  $n = 3$  case for which even the tilt running has complete freedom. The runs for  $n = 2$  and  $n = 3$  nicely converged and constitute the main result of this work. Note also that the middle-right and lower-right graphs in Fig. 1 are compatible with each other in the following sense: though  $H'_*$  may not reach zero under our assumption, the quantity  $H'_* / H_*$  may be

arbitrarily small if  $H_*$  is allowed to be arbitrarily small, too. Thus, for cases  $n = 2, 3$  when  $H_*$  is not suppressed at zero argument,  $H'_* / H_*$  is not suppressed there, too.

The probability distribution for combinations of  $H_*$ ,  $H'_*$  and  $H''_*$  are robust in the sense that they do not change significantly when one extra free parameter  $H'''_*$  is included: this indicates that they are directly constrained by the data. We tried to include an additional parameter  $(H'''_*/H_*)(H'_*/H_*)^2 m_P^6$ , but then our Markov Chains did not converge even after accumulating of the order of  $10^5$  samples. We conclude that current data do not have the sensitivity required to constrain  $H(\phi)$  beyond its third derivative and to establish the validity of the slow-roll approximation beginning from this order. On the other hand, the first two slow-roll parameters  $\epsilon(\phi) = H'^2 m_P^2 / 4\pi H^2$  and  $\tilde{\eta}(\phi) = H'' m_P^2 / 4\pi H$  are really small over the observed range (tilde is used here to avoid mixing with the conformal time  $\eta$ ). The next parameter  $\xi \equiv {}^2\lambda_H = H''' H' m_P^4 / (4\pi)^2 H^2$  is also small,  $\sim 0.01$ , though being of the order of  $\epsilon$  and  $|\tilde{\eta}|$ , not  $\epsilon^2$  or  $\tilde{\eta}^2$  as would follow from the standard slow-roll expansion. This smallness explains why our results for these parameters are similar to those obtained for the same background  $H(\phi)$  but using the slow-roll approximation to calculate the power spectra [21] (and to those in [24], too) although some important differences exist.

*Results for  $V(\phi - \phi_*)$ .* We further processed our  $n = 1, 2, 3$  runs in order to reconstruct the inflaton potential. For each run, we kept only 68% or 95% of the models with the best likelihood, and computed the corresponding inflaton potentials using Eq. (2). Note that the problem is fully symmetric under the reflection  $(\phi - \phi_*) \leftrightarrow -(\phi - \phi_*)$ . We choose to focus on one half of the solutions, corresponding to  $\dot{\phi} > 0$  and hence  $V'_* > 0$ . Our results are shown in Fig. 2. They appear to be compatible with those of Ref. [14], although a detailed comparison is difficult: first, the current method is more conservative, and second, a given order in the Taylor-expansion of  $H(\phi - \phi_*)$  is not equivalent to another order in that of  $V(\phi - \phi_*)$ . Our results are also difficult to compare with those of Ref. [26], since these authors choose to present their full allowed potentials extrapolated till the end of inflation: in principle, our Fig. 2 can be seen as a zoom on the directly constrained, small  $\phi$  region in their Fig. 2.

Our results could give the wrong impression that all preferred potentials are concave. This comes from the fact that in the representation of Fig. 2, many interesting potentials are hidden, since they almost reduce to the point  $(V_*, \Delta\phi) \rightarrow (0, 0)$ . Indeed, as long as the tensor-to-scalar ratio is not bounded from below, many low-energy inflationary models with very small  $H_*$  and  $H'_*$  (and hence tiny variation of the inflaton field during the observable e-folds) are perfectly compatible with observations. It is straightforward to show that models leading to  $n_S < 1$  and small  $r$  correspond to convex potentials

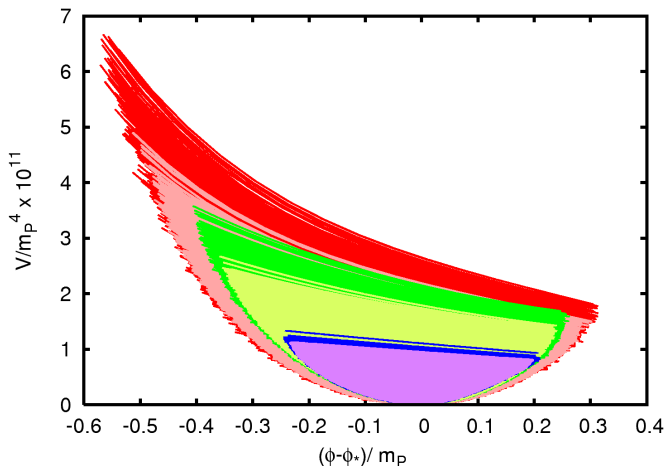


FIG. 2: Allowed inflationary potentials  $V(\phi - \phi_*)$  inferred from each of our  $n = 1, 2, 3$  runs. For each case, the light colour corresponds to models allowed at the 68% confidence levels, and the dark colour to the 95% level. The inner (bluish) region is obtained for  $n = 1$ , the intermediate (greenish) one for  $n = 2$  and the outer (reddish) one for  $n = 3$ . Each potential is plotted between the two values  $\phi_1$  and  $\phi_2$  corresponding to Hubble exit for the limits of the observable range  $[k_1, k_2] = [2 \times 10^{-4}, 0.1] \text{ Mpc}^{-1}$ : so we only see here the actual observable part of each potential. Note that this figure shows only one half of the possible solutions: the other half is obtained by reflection around  $\phi = \phi_*$ .

(like e.g. new inflation with  $V = V_0 - \lambda\phi^n$ , or one-loop hybrid inflation with  $V = V_0 + \lambda \ln \phi$ ), while models with same  $n_S$  and larger  $r$  derive from concave potentials (like e.g. monomial inflation  $V = \lambda\phi^\alpha$ ). Current data favor  $n_S < 1$ , and the upper bound on  $r$  is too loose for differentiating between these two situations. So, our allowed potentials can be split in two subsets: low-energy convex potentials and high-energy concave potentials, as illustrated in Fig. 3, in which we rescaled all allowed potentials to the same variation in  $V$  and  $\phi$ . More generally, this large degeneracy in potential reconstruction reflects the fact that an infinitely precise measurement of the scalar spectrum  $\mathcal{P}_{\mathcal{R}}$  would only constrain the function

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{4H^4}{m_P^4 H^2} \Big|_{k=aH} \quad (5)$$

(in the slow-roll approximation). This is not sufficient for inferring the correspondence between  $k$  and  $\phi$ , and hence for a unique determination of  $H(\phi)$  and  $V(\phi)$ . It is necessary to measure also the tensor spectrum, equal to

$$\mathcal{P}_h(k) = \frac{16H^2}{\pi m_P^2} \Big|_{k=aH} \quad (6)$$

in the same approximation, in order to diminish this degeneracy (see the related discussion in Ref. [37]). In the slow-roll approximation, the knowledge of  $\mathcal{P}_h(k)$  leads

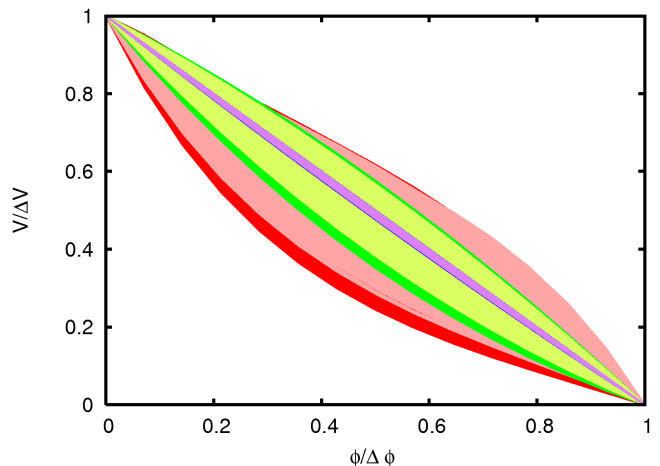


FIG. 3: Allowed inflationary potentials  $V(\phi - \phi_*)$  with the same colour/shade code as in Fig. 2, but a different choice of axes: each potential is now rescaled to the same variation in  $V$  and  $\phi$  space. This shows that many allowed potentials are actually convex. The outer region still corresponds  $n = 3$ , the intermediate one to  $n = 2$  and the inner (quasi-linear) one to  $n = 1$ .

to the unambiguous determination of  $H(\phi)$ . However, the question how unique the determination of  $H(\phi)$  is, even from *both*  $\mathcal{P}_{\mathcal{R}}(k)$  and  $\mathcal{P}_h(k)$  in the generic case beyond slow-roll, is still open because of the existence of many  $H(\phi)$  leading to the same perturbation spectra which may not be obtained from the slow-roll expansion at all [38]. Still, since the difference of these additional solutions from slow-roll ones is, in some sense, exponentially small for small slow-roll parameters, their existence might appear not significant from the observational point of view.

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  - [42] Both  $\ln A$  (the scalar amplitude) and  $n_S - 1$  (the scalar tilt deviation from one) are bounded by the data. In the  $n = 1$  case, these two quantities derive from  $H_*$  and  $H'_*$ , which are hence both constrained independently of each other.